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Procedia Computer Science 47 (2015) 319 – 325

Procedia Computer Science

On Intuitionistic Fuzzy Semi - Supra Open Set and Intuitionistic Fuzzy Semi - Supra Continuous Functions

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Abstract

In this paper, we introduce and investigate a new class of sets and functions between topological space called intuitionistic fuzzy semi-supra open set and intuitionistic fuzzy semi-supra open continuous functions respectively..

Peer-review under responsibility of organizing committee of the Graph Algorithms, High Performance Implementations and Applications (ICGHIA2014)

Keywords: Supra topological spaces: intuitionistic fuzzy supra-topological spaces: intuitionistic fuzzy semi-supra open set

1. Introduction and preliminaries.

Intuitionistic fuzzy set is defined by Atanassov [2] as a generalization of the concept of fuzzy set given by Zadesh [9]**.** Using the notation of intuitionistic fuzzy sets, Coker [3] introduced the notation of intuitionistic fuzzy topological spaces. The supra topological spaces and studied s-continuous functions and s^*-continuous functions were introduced by A. S. Mashhour [5] in 1993. In 1987, M. E. Abd El-Monsef et al. [1] introduced the fuzzy supra topological spaces and studied fuzzy supra continuous functions and obtained some properties and characterizations. In1996, Keun Min [8] introduced fuzzy s-continuous, fuzzy s-open and fuzzy s-closed maps and established a number of characterizations. In 2008, R. Devi et al [4] introduced the concept of supra α-open set , and in 1983, A. S. Mashhour et al. introduced, the notion of supra- semi open set, supra semi-continuous functions and studied some of the basic properties for this class of functions. In 1999, Necla Turan [6] introduced the concept of intuitionistic fuzzy supra topological space . In this paper, we introduce the notation of intuitionistic fuzzy semi-supra open sets and the basic properties of intuitionistic fuzzy semi-supra open sets and introduce the notation of intuitionistic

1

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Peer-review under responsibility of organizing committee of the Graph Algorithms, High Performance Implementations and Applications (ICGHIA2014)
doi:10.1016/j.procs.2015.03.211

fuzzy semi-supra continuous functions.

Throughout this paper, by ($X,\tau)$ or simply by X we will denote the intuitionistic fuzzy supra topological space (briefly, IFTS). For a subset A of a space ($X,\tau)$, cl(A), int(A) and $\overline{A}$ denote the closure of A, The interior of A and the complement of A respectively. Each intuitionistic fuzzy supra set (briefly, IFS) which belongs to ($X,\tau)$ is called an intuitionistic fuzzy supra open set (briefly, IFSOS) in X. The compliment $\overline{A}$ of an IFSOS A in X is called an intuitionistic fuzzy supra closed set (IFSCS) in X.

We introduce some basic notations and results that are used in the sequel.

Definition 1.1 [2] Let X be a non empty fixed set and I be the closed interval [0,1]. In intuitionistic fuzzy set (IFS) A is an object of the following form

$$A=\{\langle x,\mu_A(x),\nu_A(x)\rangle : x\in X\}$$

where the mapping $\mu_A : X \to I$ *and* $\nu_A : X \to I$ denote the degree of membership (namely $\mu_A(x))$ and the degree of non membership (namely $\nu_A(x))$ for each element $x\in X$ to the set A, respectively, and $0\le \mu_A(x)+\nu_A(x)\le 1$ for each $x\in X$.

Obviously, every fuzzy set A on a nonempty set X is an IFS of the following form

$A=\{\langle x,\mu_A(x),1\text{-}\ \mu_A(x)\rangle : x\in X\}$

Definition 1.2 [2] Let A and B be IFSs of the form $A=\{\langle x,\mu_A(x),\nu_A(x)\rangle : x\in X\}$ *and* $A=\{\langle x,\mu_B(x),\nu_B(x)\rangle : x\in X\}$ and $B=\{\langle x,\mu_B(x),\nu_B(x)\rangle : x\in X\}$. Then

(i) $A\subseteq B$ *if and only if* $\mu_A(x)\le\mu_B(x)$ *and* $\nu_A(x)\ge\nu_B(x)$;

(ii) $\overline{A}=\{\langle x,\nu_A(x),\mu_A(x)\rangle : x\in X\}$;

(iii) $A\cap B=\{\langle x,\mu_A(x)\wedge\mu_B(x),\nu_A(x)\vee\nu_B(x)\rangle : x\in X\}$;

(iv) $A\cup B=\{\langle x,\mu_A(x)\vee\mu_B(x),\nu_A(x)\wedge\nu_B(x)\rangle : x\in X\}$;

(v) A = B iff $A\subseteq B$ *and* $B\subseteq A$;

(vi) [ ] A $=\{\langle x,\mu_A(x),1\text{-}\ \mu_A(x)\rangle : x\in X\}$;

(vii) $\langle\rangle A=\{\langle x,1-\nu_A(x),\ \nu_A(x)\rangle : x\in X\}$;

(viii) $1_\sim=\{\langle x,1,0\rangle, x\in X\}$ and $0_\sim=\{\langle x,1,0\rangle, x\in X\}$;

We will use the notation $A=\langle x,\mu_A,\mu_A\rangle$ instead of A= $\{\langle x,\mu_A(x),\ \nu_A(x)\rangle : x\in X\}$;

Definition 1.3. [6] A family τ of IFS's on X is called an intuitionistic fuzzy supra topology (IFST for short) on X if $0_\sim\in\tau, 1_\sim\in\tau$ *and* τ is closed under arbitrary suprema. Then we call the pair (X,τ) an intuitionistic fuzzy supra topological space (IFSTS for short). Each member of τ is called an intuitionistic fuzzy supra open set and the complement of an intuitionistic fuzzy supra open set is called an intuitionistic fuzzy supra closed set. The intuitionistic fuzzy supra closure of IFS A is denoted by s-cl(A). Here s-cl(A) is the intersection of all intuitionistic fuzzy supra closed sets containing A. The intuitionistic fuzzy supra interior of A will be denoted by s-int(A). Here, s-int(A) is the union of all intuitionistic fuzzy supra open sets contained in A.

Definition 1.4. [7] Let (X,τ) be an intuitionistic fuzzy supra topological space. An IFS A $\in IF(X)$ is called

(a) intuitionistic fuzzy semi-supra open iff A $\subseteq$ s-cl(s-int(A))

(b) intuitionistic fuzzy α- supra open iff A $\subseteq$ s-int(s-cl(s-int(A)))

(c) intuitionistic fuzzy pre-supra open iff A $\subseteq$ s-int(s-cl(A))

Let f be a mapping from an ordinary set X into an ordinary set Y, if

B = $\{\langle y, \mu_B(y),\ \nu_B\ (y)\rangle : y \in Y\ \}$ is an IFST in Y, then the inverse image of B under f is an IFST defined by $f^{-1}(B) = \ \{\langle x, f^{-1}(\mu_B)(x), f^{-1}(\nu_B)(x)\ \rangle : x \in X\ \}$

The image of IFST A = $\{\ \langle y, \mu_A(y),\ \nu_A\ (y)\ \rangle : y \in Y\ \}$ under f is an IFST defined by

$f(A) = \{\ \langle y, f(\mu_A)(y),\ f(\nu_A)(y)\ \rangle : y \in Y\ \}.$

2. Intuitionistic fuzzy semi-supra open set.

Definition 2.1. Let (X,τ) be an IFS topological space. An intuitionistic fuzzy set A is called an intuitionistic fuzzy semi-supra open set (briefly IFSSOS) if A $\subseteq$ s-cl(s-int(A)). The complement of an intuitionistic fuzzy semi-supra open set is called an intuitionistic fuzzy semi-supra closed set.

Theorem 2.2. Every intuitionistic fuzzy supra open set is intuitionistic fuzzy semi-supra open set

Proof: Let A be an intuitionistic fuzzy supra open set in (X,τ). Since A $\subseteq$ s-cl(A) we get A $\subseteq$ s-cl(s-int(A)) then s-int(A) $\subseteq$ s-cl(s-int(A)). Hence A $\subseteq$ s-cl(s-int(A)).

The converse of the Theorem 2.2 need not be true as shown by the following example.

Example 2.3 Let $X = \{a, b\}$, $A = \{x, \langle 0.2, 0.4\rangle, \langle 0.5, 0.6\rangle\}$, $B = \{x, \langle 0.6, 0.2\rangle, \langle 0.3, 0.4\rangle\}$, and $C = \{x, \langle 0.3, 0.4\rangle, \langle 0.4, 0.4\rangle\}$, $\tau = \{0_\sim, 1_\sim, A, B, A \cup B\}$. Then C is called intuitionistic fuzzy semi -supra open but not intuitionistic fuzzy supra open set.

Theorem 2.4. Every intuitionistic fuzzy α supra open is intuitionistic fuzzy semi- supra open

Proof: Let A be an intuitionistic fuzzy α supra open in (X,τ), then $A \subseteq s-\text{int}(s-cl(s-\text{int}(A))).$ It is obvious that $s-\text{int}(s-cl(s-\text{int}(A))) \subseteq s-cl(s-\text{int}(A)).$ Hence $A \subseteq s-cl(s-\text{int}(A)),$

The converse of the Theorem 2.4 need not be true as shown by the example.

Example2.5

Let $X = \{a, b\}$, $A = \{x, \langle 0.2, 0.3\rangle, \langle 0.5, 0.3\rangle\}$, $B = \{x, \langle 0.1, 0.2\rangle, \langle 0.6, 0.5\rangle\}$ and $C = \{x, \langle 0.2, 0.3\rangle, \langle 0.2, 0.3\rangle\}$ $\tau = \{0_\sim, 1_\sim, A, B, A \cup B\}$. Then C is called intuitionistic fuzzy semi-supra open but not intuitionistic fuzzy α -supra open set.

Theorem 2.6. Every intuitionistic regular supra open set is intuitionistic fuzzy semi -supra open set

Proof: Let A be an intuitionistic fuzzy regular supra open set in (X,τ). Then A $\subseteq$ (s-cl(A)). Hence A $\subseteq$ s-cl(s-int(A)).

The converse of the Theorem2.6 need not be true as shown by the following example.

Example2.7

Let $X=\{a,b\}$, $A=\{x,\langle 0.2,0.3\rangle,\langle 0.5,0.3\rangle\}$, $B=\{x,\langle 0.1,0.2\rangle,\langle 0.6,0.5\rangle\}$ and $C=\{x,\langle 0.2,0.3\rangle,\langle 0.2,0.3\rangle\}$ $\tau=\{0_{\sim}, 1_{\sim}, A, B, A\cup B\}$. Then C is intuitionistic fuzzy semi- supra open but not intuitionistic fuzzy regular -supra open set.

Theorem 2.8.

i) Arbitrary union of intuitionistic fuzzy semi- supra open sets is always intuitionistic fuzzy semi- supra open set.

ii) Finite intersection of intuitionistic fuzzy semi- supra open sets may fail to be intuitionistic fuzzy semi- supra open set.

iii) $1_{\sim}$ is an intuitionistic fuzzy semi- supra open set.

Proof: (i) Let $\{A_\lambda : \lambda \in \wedge\}$ be a family of intuitionistic fuzzy semi supra open set in a topological space X. Then for any $\lambda \in \wedge$,we have $A_\lambda \subseteq$ s-cl(s-int(A_λ))

Hence $U_{\lambda\in\wedge}\ A_\lambda \subseteq U_{\lambda\in\wedge}$ (s-cl(s-int(A_λ))

$\subseteq$ s-cl($U_{\lambda\in\wedge}$ (s-int(A_λ))

$\subseteq$ s-cl(s-int($U_{\lambda\in\wedge}$ (A_λ))

Therefore $U_{\lambda\in\wedge}\ A_\lambda$ is an intuitionistic fuzzy semi supra open set

Let $X=\{a,b\}, A=\{x,\langle 0.2,0.3\rangle,\langle 0.2,0.3\rangle\}, B=\{x,\langle 0.3,0.4\rangle,\langle 0.4,0.4\rangle\}$ *and*

$\tau=\{0_{\sim}, 1_{\sim}, A, B, A\cup B\}$

Hence A and B are intuitionistic fuzzy semi supra open but A$\cap$B is not intuitionistic fuzzy semi supra open set.

Theorem 2.9

(i) Arbitrary intersection of intuitionistic fuzzy semi- supra closed sets is always intuitionistic fuzzy semi- supra closed set.

(ii) Finite union of intuitionistic fuzzy semi- supra closed sets may fail to be intuitionistic fuzzy semi- supra closed set.

(iii) $0_{\sim}$ is an intuitionistic fuzzy semi- supra closed set

Proof: (i) The proof follows immediately from Theorem 2.8.

(ii)Let $X = \{a, b\}$, $A = \{x, \langle 0.2, 0.4\rangle, \langle 0.5, 0.6\rangle\}$, $B = \{x, \langle 0.6, 0.2\rangle, \langle 0.3, 0.4\rangle\}$ and $\tau = \{0_{\sim}, 1_{\sim}, A, B, A \cup B\}$ and $C = \{x, \langle 0.4, 0.4\rangle, \langle 0.3, 0.4\rangle\}$ and $D = \{x, \langle 0.2, 0.3\rangle, \langle 0.2, 0.3\rangle\}$. Hence C and D are intuitionistic fuzzy semi- supra closed but $C \cup D$ is not an intuitionistic fuzzy semi supra closed set.

Definition:2.10 The intuitionistic fuzzy semi- supra closure of a set A is denoted by semi-s-cl(A) = $\bigcup\{ G : G$ is an intuitionistic fuzzy semi- supra open set in X and $G \subseteq A \}$ and the intuitionistic fuzzy semi- supra interior of a set A is denoted by

semi-s-int(A) $= \bigcap \{G : G \text{ is a intuitionistic fuzzy semi-supra closed set in X and } G \supseteq A\}$.

Remark 2.11

It is clear that semi-s-int(A) is an intuitionistic fuzzy semi- supra open set and semi-s-cl(A) is an intuitionistic fuzzy semi- supra closed set.

Theorem 2.12

(i) X- semi-s-int(A) = semi s-cl $(X - A)$

(ii) X-semi-s-cl(A) = semi s-int $(X - A)$

(iii) if A $\subseteq B$ then semi-s-cl(A) $\subseteq$ semi-s-cl(B) and semi-s-int(A) $\subseteq$ semi-s-int(B)

Proof:

It is obvious

Theorem 2.13

i) semi-s-int(A) $\bigcup$ semi-s-int(B) $\subseteq$ semi-s-int(AUB)
ii) semi-s-int(A $\bigcap B) \subseteq$ semi-s-int(A) $\bigcap$ semi-s-int(B)
iii) if $A \subseteq B$, then semi-s-cl(A) $\subseteq$ semi-s-cl(B) and semi-s-int(A) $\subseteq$ semi-s-int(B)

Proof: It is obvious.

Theorem 2.14

i) The intersection of an intuitionistic fuzzy supra open set and an intuitionistic fuzzy semi- supra open set is an intuitionistic fuzzy semi- supra open set.
ii) The intersection of an intuitionistic fuzzy semi- supra open set and an intuitionistic fuzzy pre- supra open set is an intuitionistic fuzzy pre- supra open set.

Proof: It is obvious.

3. Intuitionistic fuzzy semi-supra continuous map

Definition 3.1. Let (X, τ) and (Y, σ) be two intuitionistic fuzzy semi- supra open sets and μ be an associated supra topology with τ .A map $f : (X, \tau) \to (Y, \sigma)$ is called intuitionistic fuzzy semi- supra continuous map if the inverse image of each open set in Y is an intuitionistic fuzzy semi- supra open in X.

Theorem 3.2. Every intuitionistic fuzzy supra continuous map is intuitionistic fuzzy semi-supra continuous map.

Proof: Let $f:(X,\tau)\to(Y,\sigma)$ be an intuitionistic fuzzy supra continuous map and A is an open set in Y. Then f^{-1}(A) is an open set in X. Since μ is associated with τ. Then $\tau\subseteq\mu$.Therefore f^{-1}(A) is an intuitionistic fuzzy supra open set in X which is an intuitionistic fuzzy supra open set in X. Hence f is an intuitionistic fuzzy semi-supra continuous map.

Remark:3.3 Every intuitionistic fuzzy semi-supra continuous map need not be intuitionistic fuzzy supra continuous map.

Theorem:3.4 Let (X,τ) and (Y,σ) be two topological spaces and μ be an associated supra topology with τ. Let f be a map from X into Y. Then the following are equivalent.

i) f is an intuitionistic fuzzy semi-supra continuous map.

ii) The inverse image of a closed sets in Y is an intuitionistic fuzzy semi closed set in X.

iii) Semi-s-cl(f^{-1}(A)) $\subseteq f^{-1}$(cl(A)) for every set A in Y.

iv) f (semi-s-cl(A)) $\subseteq$cl(f(A)) for every set A in X.

v) f^{-1}(int(B)) $\subseteq$ semi-s-int(f^{-1}(B)) for every set B in Y.

Proof: (i)$\Rightarrow$(ii): Let A be a closed set in Y. Then Y$-$A is open in Y, Thus f^{-1}(X$-$A) =X$-f^{-1}$(A) is semi open in X. It follows that f^{-1}(A) is a semi-s closed set of X.

(ii)$\Rightarrow$(iii): Let A be any subset of X. Since cl(A) is closed in Y then it follows that f^{-1}(cl(A)) is semi-s closed in X. Therefore, f^{-1}(cl(A)) = semi-s- cl(f^{-1}(cl(A)) $\supseteq$ semi-s-cl(f^{-1}(A))

(iii)$\Rightarrow$(iv): Let A be any subset of X. By (iii) we obtain f^{-1}(cl(f((A))) $\supseteq$ semi-s-cl($f^{-1}(f$(A)))$\supseteq$ semi-s-cl (A) and hence f(semi-s-cl(A)) $\subseteq$cl(f(A)).

(iv)$\Rightarrow$(v): Let f (semi-s-cl(A)) $\subseteq$f(cl(A) for every set A in X. Then semi-s-cl(A)) $\subseteq f^{-1}$(cl($f(A)$)

X$-$semi-s-cl(A) $\supseteq$X$-f^{-1}$(cl($f(A)$)) and semi-s-int(X$-$A) $\supseteq f^{-1}$ (int(Y$-$f(A))).Then semi-s-int(f^{-1}(B)) $\supseteq f^{-1}$ (int(B)). Therefore f^{-1} (int(B)) $\subseteq$s-int(f^{-1}(B)) for every B in Y.

(v)$\Rightarrow$(i): Let A be an open set in Y. Therefore f^{-1} (int(A)) $\subseteq$semi-s-int(f^{-1}(A)), hence f^{-1}(A) $\subseteq$ semi-s-int(f^{-1}(A). But we know that semi-s-int(f^{-1}(A) $\subseteq f^{-1}$(A), then f^{-1}(A) = semi-s-int(f^{-1} (A)). Therefore

f^{-1}(A) is a semi-s-open set.

Theorem:3.5 If a map $f:(X,\tau)\to(Y,\sigma)$ is a semi-s continuous and g:$(y,\sigma)\to(z,\eta)$is continuous, Then $g\circ f$ is semi-s-continuous.

Proof: Obvious.

Theorem:3.6 Let a map $f:(X,\tau)\to(Y,\sigma)$ be an intuitionistic fuzzy semi-supra continuous map, then one of the following holds

i) f^{-1}(semi-s-int(A)) $\subseteq$ int(f^{-1}(A)) for every set A in Y.

ii) cl(f^{-1}(A)) $\subseteq$ f^{-1}(semi-s-cl(A)) for every set A in Y.

iii) f(cl(B)) $\subseteq$ semi-s-cl(f(B)) for every set B in X.

Proof: Let A be any open set of Y, then condition (i) is satisfied, then f^{-1}(semi-s-int(A)) $\subseteq$ int(f^{-1}(A)). We get, f^{-1}(A) $\subseteq$ int(f^{-1}(A)) . Therefore f^{-1}(A) is an intuitionistic fuzzy supra open set. Every intuitionistic fuzzy supra open set is an intuitionistic fuzzy semi supra open set. Hence f is an intuitionistic fuzzy semi- s continuous function. If condition (ii) is satisfied, then we can easily prove that f is an intuitionistic fuzzy semi -s continuous function if condition (iii) is satisfied , and A is any open set of Y. Then f^{-1}(A)is a set in X and f(cl(f^{-1}(A)) $\subseteq$ semi-s-cl(f(f^{-1}(A))). This implies f (cl(f^{-1}(A))) $\subseteq$ semi-s-cl(A). This is nothing but condition (ii) . Hence f is an intuitionistic fuzzy semi -s continuous function.

References

1. M.E. Abd El-monsef and A.E.Ramadan, On fuzzy supra topological spaces, Indian J. Pure and Appl.Math.no.4, 18(1987), 322-329
2. K.T. Atanassov, Intuitionistic fuzzy sets, Fuzzy sets and systems, 20(1986),87-96.
3. D. Coker, An introduction to Intuitionistic fuzzy topological spaces, Fuzzy sets and systems, 88 (1997) 81-89
4. R. Devi, S. Sampathkumar and M. Caldas, On supra α-open sets and supra α-continuous functions, General Mathematics, Vol.16,Nr.2 (2008),7784.
5. A.S. Mashhour, A.A. Allam,F.H.Khedr,On supra topological spaces, IndianJ.Pure and Appl.Math.no.4, 14 (1983), 502-510
6. N. Turanl, On intuitionistic fuzzy supra topological spaces, international conference on modeling and simulation, spain, vol II, (1999) 69-77.
7. N. Turanl, An over view of intuitionistic fuzzy supra topological spaces, Hacettepe Journal of mathematics and statistics, Volume 32 (2003),17-26.
8. Won Keun Min,On fuzzy s-continuous functions, Kangweon-Kyungki Math.J. no.1, 4(1996),77-82.
9. L.A. Zadeh, Fuzzy sets, Information and control, 8 (1965), 338-353.